The Role of Credit Rationing and Collateral in Debt Financing[†]

Eike Houben[*] and Peter Nippel[**]

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[*] Dipl.-Vw. Eike Houben, Christian-Albrechts-Universität zu Kiel, Institut für Betriebswirtschaftslehre, Lehrstuhl für Finanzwirtschaft, Olshausenstraße 40, 24098 Kiel; houben@bwl.uni-kiel.de

[**] Prof. Dr. Peter Nippel, Christian-Albrechts-Universität zu Kiel, Institut für Betriebswirtschaftslehre, Lehrstuhl für Finanzwirtschaft, Olshausenstraße 40, 24098 Kiel; nippel@bwl.uni-kiel.de

Abstract

Credit rationing and the use of collateral are widely observed in debt financing. To our view there is yet no appropriate theoretical explanation for these facts. In the standard debt financing models the occurrence of credit rationing can be explained based on suitable assumptions. But those are by no means general. Furthermore, the use and the form of collateral is limited. In our model we show that credit rationing and the use of collateral are always necessary for debt financing if lenders are rational. We do so under less strict assumptions which are, to our understanding, much more realistic than those typical for standard adverse selection or moral hazard models. We assume that the borrower's opportunity set is "unbounded", at least from the viewpoint of the lender. This means that no arbitrary restrictions are imposed on the set of possible distributions of future cash flow from which the borrower can unobservably choose one. As a result a rational lender granting a pure debt should never take any risk, neither an exogenous one resulting from the project nor a an endogenous one resulting from the information asymmetry. Furthermore, we extend the set of possible collateral to property rights over physical and non physical assets, and explain how a superior lender's information can work as collateral.

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1 Introduction

One of the obvious facts about finance is that investment projects are usually not completely financed with debt. This can be interpreted as a result of credit rationing in the sense that a borrower cannot receive debt up to the required investment even if the project has a positive NPV. If the financial gap cannot be filled with equity even an advantageous project will be dropped.

Since in perfect markets debt would be available up to the present value of expected future cash flows, credit rationing has to be explained with market imperfections. It can be shown that the purpose of credit rationing is to avoid the "endogenous" risk resulting from informational asymmetries between borrower and lender. But, there is no reason why a lender should not take some of the "exogenous" investment risk if he is compensated for this risk by a higher lending rate. Furthermore, there is no need for the widely observed use of collateral which is explained in the banking literature by the risk avoidance preference of the lender.

To our knowledge, there still exists no robust model explaining the regular occurrence of credit rationing together with the use of collateral. The well known models of debt financing by Stiglitz and Weiss (1981) and others in the same tradition can only prove that there may be credit rationing in equilibrium. But, since under slightly different assumptions credit rationing does not occur, no explanation can be given for the fact that lenders almost always ration credit. Furthermore, the use of collateral can, but need not be necessary as a mean to influence the borrower's actions or to obtain information about the investment project in a screening process as explained by Bester (1985) and Bester and Hellwig (1987). In addition, the only sensible form of collateral in the Stiglitz and Weiss (1981) framework are sureties. With regard to the stylized facts this is not very satisfactory and further weakens the explanation ability of these models.

A second class of models explaining the occurrence of credit rationing and the use of collateral are the so called "diversion"-models by Hart and Moore (1994) and Hart and Moore (1998). These models are not based on the assumption of an information asymmetry between the borrower and the lender but assume that the borrower can divert the whole cash flow from the project into his "private pocket". Thus, collateral in form of liquidation rights over assets in case of default is needed in order to protect the lender against dilution. Otherwise the lender would never pay back any of the debt. Since in a renegotiation the borrower can drive the repayment amount down to the liquidation value of the assets, a rational lender does not lend more than this liquidation value. Hence, credit rationing can occur if the liquidation value of an asset is smaller than its procurement costs, which is usually the case. However, this result is only valid if the borrower has all the bargaining power. If this assumption is dropped, credit rationing and the covering of the whole debt by collateral are no longer compulsory results of the models.

In our view, the assumption of an informational asymmetry is more realistic than the assumption of an unlimited diversion ability. Hence, we analyze an extended moral hazard setting to proof theoretically that credit rationing does not only occur in some special equilibria but follows necessarily from an informational disadvantage of the lender. Our theory leads to a central role of collateral, especially in the form of a limitation of the borrower's property rights over the firm's assets.

Building on a debt financing model in the tradition of Stiglitz and Weiss (1981), we explain credit rationing and the use of collateral as a rational and always necessary reaction of lenders to an extreme form of information asymmetry. To make our contribution to the theory clear we start with a short look at debt financing in the neoclassical world. Afterwards we show that in traditional adverse selection and moral hazard models credit rationing may or may not occur in equilibrium. The occurrence depends on the assumptions made about the set of projects searching for funding or the set of projects the borrower can choose from, respectively. To overcome this unsatisfactory ambiguity in the established theory we introduce a more realistic assumption on the informational structure: We assume that in a typical moral hazard situation it is not possible for the lender to delimit, in his perception, the borrower's opportunity set. As a consequence he will rationally limit his debt offer rigidly. In this context collateral is used to restrict the borrower's actions and furthermore to derive information in a screening process.

To our knowledge the only other paper analyzing a financing decision in the presence of an unbounded entrepreneur's opportunity set is the one by Ravid and Spiegel (1997). In their security design framework the authors come to the conclusion that in this situation the optimal financing contract should be a combination of common equity and secured debt. In contrast to them we restrict ourselves to a deeper analysis of debt financing.

In the last part of the paper we briefly analyze the situation in which also the lender has private information useful to asses the profitability of the investment project. In our estimation, this assumption is realistic since banks or venture capital firms may have such information due to their experience. We show that the need for collateral can then be (partly) substituted by the lender's information.

2 Credit rationing and collateral in classical debt financing models

2.1 Neoclassic models

One of the main characteristics of neoclassical finance models is the non-existence of any informational asymmetries. Borrower and lender both know the distributions of future cash flow for all possible projects and the borrower's project choice is observable and verifiable. Then, they will agree upon which project to choose and specify this in the debt contract. There is no moral hazard problem because of the verifiability of the borrower's action.

The contractual lending rate must increase in the financial leverage. That is because the lender bears more risk, and hence asks for a higher risk premium. All projects with a positive net present value could be completely financed with debt if the interest rate is high enough so that the lender receives up to the total cash flow in each state of the world. Nothing more can be gained by the use of collateral. The use of collateral would only lead to a reallocation of risk if more than one lender is present. The only form of "credit rationing" under neoclassical conditions is thus the rejection of unprofitable investment projects. However, this is not really credit rationing in any sensible meaning.

The above argumentation can be formalized in the following simple model. The agreed upon investment project requires an investment of I in t_0 and leads to an uncertain cash flow of R in t_1 where

$$R = \begin{cases} R^{+} & \text{with probability } p \\ R^{-} & \text{with probability } (1-p) \end{cases}$$

The risk-free interest rate in the economy is denoted by i, and the contracted lending rate by r. Assume that all individuals are risk-neutral. The lender provides (a part of) the capital needed to start the project if in doing so he is not worse off than investing his money at the risk-free rate, that is if

$$p \cdot \min(R^{+}, (1+r) \cdot D) + (1-p) \cdot \min(R^{-}, (1+r) \cdot D) \geq (1+i) \cdot D \tag{1}$$

is fulfilled, where D denotes the capital supplied by the lender, that is the face value of the debt. It is easy to see that if

$$D > \frac{R^{-}}{1+i}$$

the lender has to bear some of the project's exogenous risk. However, this risk-taking is no problem as long as r can be adjusted accordingly to satisfy (1). Formally we have

$$\begin{aligned} & p \cdot (1+r) \cdot D + (1-p) \cdot R^{-} \geq (1+i) \cdot D \\ \Leftrightarrow\ & r \geq \frac{1+i}{p} - 1 - \frac{1-p}{p} \cdot \frac{R^{-}}{D} \end{aligned} \tag{2}$$

Therefore, r is strictly increasing in D, as long as $(1+r)D < R^{+}$, and decreasing in p.

The maximum amount of D the borrower can raise is limited only by the project's present value. Since the maximum repayment obligation $(1+r) \cdot D$ effectively cannot exceed R^{+}, we obtain from (1) an upper bound for D:

$$D_{\max} = \frac{p \cdot R^{+} + (1-p) \cdot R^{-}}{1+i} = \frac{E(R)}{1+i} \tag{3}$$

For this face value of debt the maximum lending rate, $r_{\max}$, is necessary to compensate the lender for the risk:

$$r_{max} = (1+i) \cdot \frac{R^+}{p \cdot R^+ + (1-p) \cdot R^-} - 1 \qquad (4)$$

D_{max} is exactly the present value of the cash flow. Up to this investment the project is profitable and could be totally financed with debt. Only if there is an exogenous cap on r lower than r_{max}, the risk-taking by the lender might be limited and thus credit rationing might occur.[1]

2.2 "Classic" information economic models

2.2.1 Hidden information models

To make debt finance non trivial some kind of information problems have to be introduced. If lenders do not know the "type" of a potential borrower but know what types exist, and there is common knowledge about the distribution of types, we have a standard hidden information model. The borrower's type has to be characterized by the cash flow distribution of the project he is seeking debt financing for.

Due to the information asymmetry between borrower and lender, credit rationing may or may not occur. It does so if we have the classical adverse selection problem: It can be profitable for some borrowers with a negative net present value project to ask for debt financing under conditions appropriate for the average project if their stake in the financing is "small enough".[2] Financing such a project leads to an expected loss for the lender. To avoid financing these projects, the lender may reduce the size of the debt contract so that projects with a negative net present value become unprofitable for the borrowers. This leads to credit rationing if the maximum debt is smaller than the required investment. If, for some reasons, equity is also rationed, financing the necessary investment for a profitable project might not be possible.

However, credit rationing does not necessarily occur in a hidden information environment. It does so only under special assumptions on cash flows and the distribution of the different types (projects). To see this we have to extent our simple model from the previous chapter. Assume that the set of possible "borrower-types" consists only of

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[1] Such a cap may, e.g., result from a regulation of the credit market.

[2] That is a result of the well known gambling incentive result by Jensen and Meckling (1976).

two types and thus only two projects have to be looked at. Assume that both require the same initial investment *I*. The uncertain cash flows resulting from the projects are

$$R_1 = \begin{cases} R_1^+ & \text{with probability } p_1 \\ R_1^- & \text{with probability } (1 - p_1) \end{cases}$$

for type 1, and

$$R_2 = \begin{cases} R_2^+ & \text{with probability } p_2 \\ R_2^- & \text{with probability } (1 - p_2) \end{cases}$$

for type 2.

Only the borrower knows his type. The lender assigns an a priori probability of q to the occurrence of the type-1-borrower, and $(1 - q)$ to the type-2-borrower.

To become active in the capital market, that is seek or offer debt financing, the following participation constraints for the lender:

$$\begin{aligned} & x \cdot q \cdot [p_1 \cdot \min(R_1^+, (1+r) \cdot D) + (1-p_1) \cdot \min(R_1^-, (1+r) \cdot D)] \\ & + y \cdot (1-q) \cdot [p_2 \cdot \min(R_2^+, (1+r) \cdot D) + (1-p_2) \cdot \min(R_2^-, (1+r) \cdot D)] \\ & \geq (1+i) \cdot D \end{aligned} \tag{5}$$

for the type-1-borrower:

$$p_1 \cdot \max(R_1^+ - (1+r) \cdot D, 0) + (1-p_1) \cdot \max(R_1^- - (1+r) \cdot D, 0) \geq (1+i) \cdot (I - D) \tag{6}$$

and the type-2-borrower:

$$p_2 \cdot \max(R_2^+ - (1+r) \cdot D, 0) + (1-p_2) \cdot \max(R_2^- - (1+r) \cdot D, 0) \geq (1+i) \cdot (I - D) \tag{7}$$

have to be fulfilled. In (5) x (y) is a dummy variable indicating that (6) ((7)) is fulfilled, $x = 1$ ($y = 1$), or not, $x = 0$ ($y = 0$). Given these two types of projects we have credit rationing in equilibrium if the riskier project is inefficient and the hidden information problem is in some sense "severe" so that the lender has to avoid financing such an inefficient project to break even in expected terms ex ante. The details are stated in the following proposition:

Proposition 1: *Assume that* $R_1^+ > R_2^+ > R_2^- > R_1^-$ *and* $E(R_1) < (1+i)\cdot I < E(R_2)$. *Then, credit rationing occurs if* $q\cdot E(R_1) + (1-q)\cdot E(R_2) - qp_1\cdot(R_1^+ - R_2^+) < (1+i)\cdot I$. *Thus, a sufficient condition for the occurrence of credit rationing is the average project being inefficient, i.e.,* $q\cdot E(R_1) + (1-q)\cdot E(R_2) < (1+i)\cdot I$.[3] *Collateral is not needed since the lender is willing to take exogenous risk. He always does so except for* $R_1^- > (1+i)\cdot D$, *or for* $R_1^- < (1+i)\cdot D < R_2^-$ *and only type-2-borrower becomes active in the market.*

Proof: *See Appendix A.*

The economic intuition behind proposition 1 is that since project 1 is inefficient, the lender has to assure that not just the type-1-borrower becomes active in the market. Due to the positive relation between *D* and *r*, resulting from the lender's participation constraint and thus a "high" risk premium for "high" debt levels, the investment project may become disadvantageous for the type-2-borrower if he asks for a "high" debt contract. Hence, the type-2-borrower may not want to raise a debt up to the required investment. Therefore, credit rationing can occur and a positive NPV project might be sacrificed if the borrower cannot raise sufficient capital from other sources.

However credit rationing is not a compulsory result of the hidden information model:

Proposition 2: *Both projects could be all debt financed if they have a positive NPV on average and do not differ too much with respect to risk, that is if:* $qE(R_1) + (1-q)E(R_2) > (1+i)\cdot I$ *and* $(R_1^+ - R_2^+)$ "*small*".

Proof: *See Appendix B.*

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[3] Furthermore, borrowers do not ask for debt of size *I* if $R_2^+ < (1+r)\cdot I$ *and* $E(R_1) < (1+i)\cdot I$.

With a positive NPV on average the lender does not care much about the borrower's type. If both borrower-types ask for credit, a high enough lending rate can be found to fulfill his participation constraint. The only case for credit rationing is given if with $D = I$ the type-2-borrower would have to pay more than all the cash flow in the good state to make the lender break even in expected terms. Then, the type-2-borrower would do without that high debt. He would "voluntarily" drop his project if he has no access to enough equity.

If $E(R_1) < (1+i)I$ there is even too much debt financing in the sense that the good type (with project 2) subsidizes the bad type and both can finance their projects completely with debt. This is kind of opposite to credit rationing.

2.2.2 Hidden action models

The standard hidden action model is close to the standard hidden information model. The difference is that we now have just one type of borrower with a set of possible investment projects. After receiving the capital, the borrower can decide which of the projects will be realized. This choice enables him to influence the cash flow. The lender does not know which project the borrower will choose. Hence, he is not aware of the distribution of the cash flow, but what he knows is the set of all possible projects. Due to this knowledge and the terms of the debt contract he can anticipate the borrower's project choice although he cannot observe it.

As in the hidden information model, the choice of an inefficient project may be advantageous for the borrower due to the gambling incentive if his capital contribution is not too "high". Thus, credit rationing may be necessary to induce the borrower to choose an efficient project. But, as will be shown in the following model, this does not mean that the lender never takes a risk.

Assume that the borrower has the choice between two investment projects with the following payoff structures

$$R_1 = \begin{cases} R_1^+ & \text{with probability } p_1 \\ R_1^- & \text{with probability } (1 - p_1) \end{cases}$$

and

$$R_2 = \begin{cases} R_2^+ & \text{with probability } p_2 \\ R_2^- & \text{with probability } (1 - p_2) \end{cases}$$

where $R_1^+ > R_2^+ > R_2^- > R_1^-$. His choice can neither be observed by the lender nor by a court and is thus the borrower's private information. Both projects require an initial investment of *I*. Again, the risk-free interest rate in the economy is denoted by *i*, the contracted lending rate by *r*, and the face value of the debt contract by *D*.

Proposition 3: *Credit rationing can only occur if* $E(R_1) < (1+i)I$, *and* $\max\left\{\frac{R_1^-}{1+i}; \frac{E(R_2) - p_1 R_1^+}{(1-p_1)(1+i)}; \frac{p_2(p_2 R_2^+ - p_1 R_1^+)}{(p_2 - p_1)(1+i)} + \frac{(1-p_2)R_2^-}{1+i}\right\} < I$. *The lender can be compensated for taking exogenous risk, collateral is not needed.*

Proof: *See Appendix C.*

The economic intuition behind proposition 3 is that it is profitable for the borrower to choose the inefficient project 1 if his stake of the invested capital is not too "big". Because he then has a risk gambling incentive induced by the payoff structure. The lender can anticipate this choice and rations the debt to induce the borrower to choose project 2.

However, credit rationing does not exclude bearing an exogenous risk by the lender. On the contrary, the lender will usually take a risk. Furthermore, there is no room for collateral in the standard hidden action model, except for sureties which in fact are nothing else but indirectly extended equity.[4]

[4] Neus (1998).

3 Credit rationing, collateral, and unbounded borrower’s opportunity set

The classical information economic models are all built on the assumption of a complete information set. This does not mean that the lender knows the exact project chosen or the type of borrower, but that he knows all possible projects or all types of borrowers and their probability of occurrence, respectively. This assumption seems unrealistic. In our view, the assumption of an “unbounded” borrower’s opportunity set is more appropriate. By unbounded we mean that at least from the lender’s perspective the borrower has the choice from an unlimited set of probability distributions of future cash flows. The only general restriction is that they all have a finite net present value. In his assessment of the borrower’s opportunity set the lender cannot exclude any such project with certainty.

To clarify our assumption let us provide some economic argumentation. The number of possible projects in the borrower’s opportunity set and thus the opportunity set itself may well be limited. However, since the borrower can influence the payoff structure with tactical management decisions the number of possible projects becomes in fact that large that we can assume it to be “unbounded. This is the case especially if the borrower can trade derivatives on the capital market because this enables him to influence the payoff structure in nearly every desired form.[5] Even if the actual opportunity set is not very broad, the lender will nevertheless be unable to identify it exactly. Hence, he has to regard many different sets as possible. This is equivalent to look at the set as if it were unbounded.

To show how the results from the classic information economic models change if an unbounded opportunity set is assumed, we consider a simple theoretical model based on the classic hidden action setting.[6]

A risk-neutral borrower contacts a lender and asks for debt to finance a risky investment project. He presents a project with the following payoff structure to the lender:

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[5] Note that the efficiency of the investment project is not influenced by the trading of arbitrage-free valued derivatives.

[6] The assumption of an unbounded opportunity set could easily be applied to the classic hidden information model, too.

$$R_1 = \begin{cases} R_1^+ & \text{with probability } p_1 \\ R_1^- & \text{with probability } (1 - p_1) \end{cases}$$

Since the borrower's opportunity set is unbounded the lender expects the borrower to possibly realize another project with

$$R_x = \begin{cases} R_x^+ & \text{with probability } p_x \\ R_x^- & \text{with probability } (1 - p_x) \end{cases}$$

To express the unbounded opportunity set, we assume that R_x^+, R_x^- and p_x cannot be specified by the lender. At least from the lender's viewpoint R_x^+, R_x^-, and p_x can be arbitrarily influenced by the borrower with respect to the following conditions:

$$E(R_x) \leq \Omega < \infty \quad \forall R_x \quad \text{and} \quad E(R_1) \leq \Omega \tag{8}$$

$$R_x^- \geq \omega > 0 \text{ and } \omega \leq R_1^- \tag{9}$$

Condition (8) expresses the fact that there are no investment projects with an infinite net present value. Condition (9) states that there is a minimum cash flow generated by the borrower's project. We will discuss this assumption below.

Lemma 1: *Due to the unbounded opportunity set the expected payment to the lender is always lower than* $(1 + i)D$ *if* $(1 + i)D > \omega$*. Hence, he will make an expected loss if debt is that high. This is valid irrespective of* r*.*

Proof: *See Appendix D.*

Lemma 1 builds on the borrower's ability to influence the cash flow by choosing a project out of his unbounded opportunity set. To maximize his utility he chooses the project with the highest cash flow in the good state of nature and the lowest possible cash flow in the bad state. Since the net present value of the project's cash flow is limited by an upper bound the borrower has to accept a low probability of success. But, by minimizing the payment to the lender in the bad state he maximizes the dilution of the lender's position and hence increases his own wealth if the total expected cash flow is constant or decreases not to much. With an unbounded opportunity set the borrower has the possibility to follow this policy. To maximize his utility he will choose a project with an expected cash flow as high as Ω and with very high risk, in the sense of default probability, such that the lender's position is worth not much more than ω.

Lemma 1 leads directly to the following proposition:

Proposition 4: *With an unbounded opportunity set the maximum debt is* $D_{\max} = \frac{\omega}{1+i}$.

Hence, the lender never takes a risk even though he is risk-neutral.

Proof: *The proof follows directly from lemma 1.*

A rational lender only provides capital for an investment project if the expected change in his wealth position is not negative. But, due to the unbounded borrower's opportunity set, the lender cannot avoid an expected loss if he takes any risk, no matter what lending rate he demands.

The only way to induce a lender to provide capital is thus to make his debt position risk-free. This can be done only by limiting the debt to the present value of the lower bound ω of the cash flow. Where does this value come from? In principle the lower bound of the cash flow is always zero if, e.g., the borrower can carry all the money into a casino. Hence, a value of $\omega > 0$ can be interpreted as the result of some collateral requirements. Besides the already mentioned sureties the main form of collateral is the limitation of the borrower's property rights over his assets. That is, the borrower gives up the right to sell an asset serving as security. Furthermore, in case of default the ownership of this asset is shifted automatically to the lender.[7] By this construction the lender is assured to get at least the liquidation value of the asset no matter

[7] In case of a trade credit the lender sells an asset usually with a reservation of proprietary rights to the borrower. That means that the borrower does not get the property right over the asset until he has fully paid for it.

which action the borrower takes. Thus, that kind of collateral serves as a mechanism to limit the borrower's space of action.

Corollary 1: *The existence of collateral of value ω is a necessary condition for debt financing. Credit rationing is likely to occur since usually* $\frac{\omega}{1+i} < I$.

An asset's liquidation value is usually smaller than its procurement costs. Thus, the needed investment expenditure cannot be financed completely with debt. In this sense credit rationing occurs. Furthermore, for running or starting a company capital is not just needed to buy physical or non physical assets but also to pay wages, rent offices etc. Obviously, these expenditures cannot work as collateral and thus cannot be financed with debt.

As stated in the next proposition, with strict credit rationing as derived above the lender has neither an incentive to hide his information nor to choose an inefficient project:

Proposition 5: *With strict credit rationing and collateral of value ω a borrower reveals his opportunity set truthfully and chooses the efficient investment project.*

Proof: *See Appendix E.*

The economic intuition behind proposition 5 is that since the lender takes no risk the borrower cannot gain by gambling. But then he has no incentive to start an inefficient project or to present the lender another project than the one he wants to pursue.

Propositions 4 and 5 raise the question why defaults on debt can be observed in reality if all lenders are rational. The simplest answer is that lenders are not rational in the sense of our model and hence take a risk to their disadvantage. This behavior may be due to the competition between lenders (banks) in the debt market. Since all lenders want to conclude a safe debt contract they compete for these over the price, that is the lending rate. With a sufficient number of competitors the profits will be low and expanding to the market of risky debt might appear to be a strategy to raise profits. However, as we have shown, this strategy leads to an expected loss.

But even collateral may not be safe. In case of sureties the default risk is simply shifted from the project to the borrower (or who ever is the guarantor). Thus, risk cannot be excluded. If collateral consists of securities in the form of rights over assets there may also be a risk for the lender. That is because an asset's liquidation value is risky. However, this risk can be reduced by a "conservative" estimation of the asset's liquidation value. Such a conservative estimation can be found in the company's balance sheet. This explains the predominance of balance sheet ratios in the credit allowance process.

4 Lender-information

From casual observations we know that not every debt contract is secured by collateral. How can this be explained in the light of our model? So far we have interpreted ω as the minimum payment resulting from the investment project which can be signaled truthfully to the lender. Thus, the natural interpretation of ω is the price of assets which can be realized on a secondary market. Using assets as securities was necessary since the borrower has a superior information about his type or the set of possible actions while the lender has no information about the project at all. However, in our view the assumption of this single-sided information asymmetry is not always appropriate.

Especially institutional investors such as banks or venture capital companies may have relevant information for the assessment of an investment project due to their experience. If such an investor infers from his information a minimum cash flow of the project collateral loses its importance. The assessment of the investor is thus a further signal about the project quality which the borrower could take into account. He should do so if the signal is undistorted. That is the lender has no incentive to under- or overstate his private information in order to raise the price of debt.[8] However, if risk avoidance is the lender's main objective, the signal is undistorted and can directly be used by the borrower to update his expectations about the profitability of his investment project. To see this, simply imagine the payoff of a bank assessing a presented investment pro-

[8] See Houben (2001) for a discussion of this problem.

ject with a specified capital need. If the bank comes to the conclusion that the project will have a minimum cash flow lower than the needed capital it will face an expected loss if it grants a higher unsecured debt. On the other hand, with debt lower than the minimum cash flow or even more by rejecting the borrowers request for debt, the bank loses a (nearly) risk-less investment opportunity. Thus, the borrower can conclude from the offered contract the minimum cash flow the bank expects. Hence, he can update his expectation about the distribution of the future cash flow.

However, the bank’s expectation about the minimum future cash flow can of course be wrong. Thus, if the bank overestimates the future minimum cash flow a debt default might appear.

5 Conclusion

In our paper we have shown that in the “classic” debt financing models credit rationing can but need not occur, depending on the specifications of the assumptions. The function of credit rationing is limited to a reaction to the endogenous risk for the lender resulting from the informational asymmetry. However, these models cannot explain convincingly the use of collateral in debt contracts. That is because credit rationing alone is in principle a sufficient answer to the problems resulting from information asymmetry. Furthermore, the lender is not prevented from taking some part of the exogenous risk.

These results are no longer valid if an unbounded opportunity set is assumed. Then, only strict credit rationing in combination with the use of collateral is the adequate answer to the generalized moral hazard problem. Credit is rationed to the minimum value of the debt contract safeguarded by the use of collateral. If the lender grants a higher credit he must expect a loss since there is always a project which improves the borrower’s position by diluting the lender’s one. The avoidance of risk is the only rational strategy for lenders in case of pure debt.

Debt defaults are mostly a consequence of the intentionally risk taking by lenders and could thus be avoided by the rational strategy described above.

6 Appendix

6.1 Appendix A

If $R_1^- > (1+i)\cdot D$ the minimum cash flow exceeds the payback obligation. Obviously the debt is not risky. Furthermore, the type-1-borrower does not seek a credit since $E(R_1) - (1+i)\cdot D < (1+i)\cdot(I-D)$ is valid by assumption.

If $R_1^- < (1+i)\cdot D < R_2^-$ and the type-1-borrower does not seek finance the minimum cash flow exceeds again the payback obligation. Obviously the debt is not risky.

Suppose now that $R_2^- < (1+i)\cdot D$ and both borrowers seek financing. To see that credit rationing then can occur we first have to calculate D_{max}, i.e. the maximum debt amount a type-2-borrower wants to raise. From (5) and (7) we obtain for the maximum and minimum lending rate

$$\frac{1+i}{q\cdot p_1 + (1-q)\cdot p_2} - \frac{q\cdot(1-p_1)\cdot R_1^- + (1-q)\cdot(1-p_2)\cdot R_2^-}{(q\cdot p_1 + (1-q)\cdot p_2)\cdot D} = 1+r \tag{A1}$$

$$\frac{p_2\cdot R_2^+ - (1+i)\cdot I}{p_2\cdot D} + \frac{1+i}{p_2} \geq 1+r \tag{A2}$$

We can now calculate D_{max} by inserting the left hand side of (A2) for 1+r in (A1). After rearranging, we obtain

$$D_{\max} = \frac{p_2\cdot[q\cdot(p_1\cdot R_2^+ + (1-p_1)\cdot R_1^-) + (1-q)\cdot E(R_2)]}{q\cdot(p_2-p_1)\cdot(1+i)} - \frac{(q\cdot p_1 + (1-q)\cdot p_2)\cdot I}{q\cdot(p_2-p_1)} \tag{A3}$$

Credit rationing occurs if

$$\begin{aligned} & D_{max} < I \\ \Leftrightarrow\ & q\cdot(p_1\cdot R_2^+ + (1-p_1)\cdot R_1^-) + (1-q)\cdot E(R_2) < (1+i)\cdot I \\ \Leftrightarrow\ & q\cdot E(R_1) + (1-q)\cdot E(R_2) - qp_1\cdot(R_1^+ - R_2^+) < (1+i)\cdot I \end{aligned} \tag{A4}$$

Since $qp_1 \cdot (R_1^+ - R_2^+)$ is positive by assumption, a negative average net present value $q \cdot E(R_1) + (1-q) \cdot E(R_2) - (1+i) \cdot I$ is sufficient for credit rationing to occur.

Q.E.D.

6.2 Appendix B

For $D = I$ the borrowers' participation constraints ((6) and (7)) are both fulfilled as long as $(1+r)D \le R_2^+$. With this upper bound for the borrowers' obligation the lender's participation constraint (5) is also fulfilled, given the distribution of borrowers asking for credit, if $qE(R_1) + (1-q)E(R_2) - qp_1(R_1^+ - R_2^+) \ge (1+i)I$. Since $(R_1^+ - R_2^+)$ is positive by assumption, the average $qE(R_1) + (1-q)E(R_2) - (1+i)I$ has to be positive and must not be less than $qp_1 \cdot (R_1^+ - R_2^+)$. This leads to the requirement of $(R_1^+ - R_2^+)$ being „small", which is the case if the variances of R_1 and R_2 do not differ much.

Q.E.D.

6.3 Appendix C

For $E(R_1) < (1 + i)I$ the lender wants to prevent the borrower from choosing project 1 since then he could not break even ex ante. The borrower's project choice depends on the payback obligation $(1 + r)D$. He chooses project 2 if

1. $(1+r) \cdot D < R_1^-$ since then the debt is risk free and the borrower cannot gain by gambling. The lending rate is equal to risk-free-rate, $r = i$, and the maximum debt for which this condition is fulfilled is

$$D_{\max}^a = \frac{R_1^-}{1+i}. \tag{A5}$$

2. $R_1^- < (1+r) \cdot D < R_2^-$ and if

$$E(R_2) - (1+r) \cdot D > p_1 \cdot R_1^+ - p_1 \cdot (1+r) \cdot D$$
$$\Leftrightarrow E(R_2) > p_1 \cdot R_1^+ + (1-p_1) \cdot (1+r) \cdot D$$

Then the debt is risk free again and $r = i$. The maximum debt for which this condition is fulfilled is

$$D_{\max}^{b} = \frac{E(R_2) - p_1 \cdot R_1^{+}}{(1-p_1)\cdot(1+i)} \tag{A6}$$

3. $R_2^{-} \leq (1+r)\cdot D < R_2^{+}$ and if[9]

$$\begin{aligned} & p_2 \cdot R_2^{+} - (1-p_2)\cdot(1+r)\cdot D > p_1 \cdot R_1^{+} - p_1 \cdot (1+r)\cdot D \\ \Leftrightarrow\ & \frac{p_2 \cdot R_2^{+} - p_1 \cdot R_1^{+}}{p_2 - p_1} > (1+r)\cdot D_{\max}^{c} \end{aligned} \tag{A7}$$

where r is derived from the lender's participation constraint

$$\begin{aligned} & p_2 \cdot (1+r)\cdot D + (1-p_2)\cdot R_2^{-} = (1+i)\cdot D \\ \Leftrightarrow\ & (1+r)\cdot D = \frac{1+i}{p_2}\cdot D - \frac{1-p_2}{p_2}\cdot R_2^{-} \quad . \end{aligned} \tag{A8}$$

Setting $(1+r)\cdot D = (1+r)\cdot D_{\max}^{c}$ in (A8) and equalizing (A7) and (A8) leads to

$$D_{\max}^{c} = \frac{p_2 \cdot (p_2 \cdot R_2^{+} - p_1 \cdot R_1^{+})}{(p_2 - p_1)\cdot(1+i)} + \frac{(1-p_2)\cdot R_2^{-}}{1+i} \tag{A9}$$

Thus, the borrower chooses project 2 if the debt does not exceed the maximum of the three value $D_{\max}^{a}, D_{\max}^{b}$ and $D_{\max}^{c}$, and project 1 otherwise. Since the choice of project 1 should be prevented credit rationing must occur if

$$\max\{D_{\max}^{a}, D_{\max}^{b}, D_{\max}^{c}\} < I$$

[9] Since $R_1^{+} > R_2^{-}$ for $p_2 \leq p_1$ the borrower chooses project 1. Condition (A7) is thus only valid for $p_2 > p_1$.

If both projects are efficient the lender does not care about the project choice. He can always break even ex ante by adjusting the lending rate accordingly to the anticipated project choice. Since he can anticipate the project choice correctly there is obviously no room for credit rationing.

Q.E.D.

6.4 Appendix D

We first have to show that there exist combinations of R_x^+, R_x^- and p_x such that the borrower prefers project *x* to project 1. He does so if[10]

$$\begin{aligned} & p_x \cdot (R_x^+ - (1+r)\cdot D) > p_1 \cdot (R_1^+ - (1+r)\cdot D) \\ & \Leftrightarrow p_x > \frac{E(R_1) - p_1\cdot(1+r)\cdot D - (1-p_1)\cdot R_1^-}{R_x^+ - (1+r)\cdot D} \end{aligned} \tag{A10}$$

By assumption Project *x* has to fulfill the following condition

$$\begin{aligned} & E(R_x) \le \Omega \\ & \Leftrightarrow p_x \le \frac{\Omega - R_x^-}{R_x^+ - R_x^-} \end{aligned} \tag{A11}$$

Hence, there is an infinite number of probabilities p_x and thus of projects of type *x* which the borrower prefers if

$$\begin{aligned} & \frac{\Omega - R_x^-}{R_x^+ - R_x^-} > \frac{E(R_1) - p_1\cdot(1+r)\cdot D - (1-p_1)\cdot R_1^-}{R_x^+ - (1+r)\cdot D} \\ & \Leftrightarrow [\Omega - R_x^-]\cdot[R_x^+ - (1+r)\cdot D] > [E(R_1) - p_1\cdot(1+r)\cdot D - (1-p_1)\cdot R_1^-]\cdot[R_x^+ - R_x^-] \end{aligned} \tag{A12}$$

Note that both sides of the inequality monotonically increase in R_x^+ but that for a sufficiently small R_x^- the left hand side of the inequality increases faster in R_x^+. That is be-

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[10] We implicitly assume $R_x^+ > (1+r)\cdot D > R_x^-$ and $R_1^+ > (1+r)\cdot D$.. This is for simplicity purpose only.

cause then $\Omega - R_x^- > E(R_1) - p_1 \cdot (1+r) \cdot D - (1-p_1) \cdot R_1^-$. Thus, there is an infinite number of probabilities satisfying (A10) and (A11). The existence of the arbitrary many projects with a higher expected residual for the borrower is guaranteed[11]

Next we have to analyze how the borrower will choose R_x^+. His maximization problem is

$$\begin{aligned} & p_x \cdot (R_x^+ - (1+r) \cdot D) \rightarrow \max_{R_x^+, R_x^-, p_x}! \\ & \Leftrightarrow \; E(R_x) - p_x \left[(1+r) \cdot D - R_x^- \right] - R_x^- \rightarrow \max_{R_x^+, R_x^-, p_x}! \\ & s.t. \\ & E(R_x) \leq \Omega \;\; and \;\; R_x^- \in [\omega; R_x^+) \end{aligned} \qquad \text{(A13)}$$

It follows immediately that the solution is

$$R_x^- \rightarrow \omega, \quad p_x \rightarrow 0, \; and \; R_x^+ \rightarrow \infty .$$

It is now easy to see that the lender faces an expected loss if (1+*i*) *D* > ω. We have

$$\begin{aligned} & p_x \cdot (1+r) \cdot D + (1-p_x) \cdot R_x^- < (1+i) \cdot D \\ & \Leftrightarrow p_x < \frac{(1+i) \cdot D - R_x^-}{(1+r) \cdot D - R_x^-} \end{aligned}$$

which is

$$0 < \frac{(1+i) \cdot D - \omega}{(1+r) \cdot D - \omega} \quad \Leftrightarrow \quad \omega < (1+i) \cdot D$$

given the above solution of the borrower's maximization problem.

Q.E.D

[11] Note that for $(1+r) \cdot D > R_1^+$ (A10) simplifies to $p_x > 0$ such that the proof of existence of p_x satisfying (A10) and (A11) is already done since $\Omega - R_x^- > 0$.

6.5 Appendix E

Without a lender taking risk, which is assured by credit rationing and collateral, the borrower has no incentive to choose an inefficient project. Formally he will only seek finance for a project if

$$\begin{gathered} E(R)-(1+i)\cdot D>(1+i)\cdot(I-D) \\ E(R)>(1+i)\cdot I \end{gathered} \tag{A15}$$

which is exactly the efficiency constraint. Furthermore, since the borrower maximizes his wealth he will choose the project with the maximum net present value. But, if the borrower chooses the efficient project anyway, there is no incentive for him to present another project the lender.

Q.E.D.

References

BESTER, H. (1985): Screening vs. Rationing in Credit Markets with Imperfect Information, *American Economic Review*, 75, 850-859.

BESTER, H. AND M. HELLWIG (1987): Moral Hazard and Equilibrium Credit Rationing. In *Agency Theory, Information, and Incentives*, edited by Bamberg, G. and K. Spremann, 135-166.

HART, O. AND J. MOORE (1994): A Theory of Debt Based on the Inalienability of Human Capital, *Quarterly Journal of Economics*, 109, 841-879.

HART, O. AND J. MOORE (1998): Default and Renegotiation: A Dynamic Model of Debt, *Quarterly Journal of Economics*, 113, 1-41.

HOUBEN, E. (2001): Venture Capital, Double-sided Moral Hazard, and Double-sided Adverse Selection, *working paper*, University of Kiel.

JENSEN, M. C. AND W. H. MECKLING (1976): Theory of the Firm: Managerial Behavior, Agency Costs, and Ownership Structure, *Journal of Financial Economics*, 3, 305-360.

NEUS, W. (1998): Kreditsicherheiten und Modelle der Kreditfinanzierung. In *Unternehmensführung und Kapitalmarkt*, edited by Franke, G. and H. Laux, 211-251.

RAVID, S. A. AND M. SPIEGEL (1997): Optimal Financial Contracts for a Start-Up with Unlimited Operating Discretion, *Journal of Financial and Quantitative Analysis*, 32, 269-286.

STIGLITZ, J. E. AND A. WEISS (1981): Credit Rationing in Markets with Imperfect Information, *American Economic Review*, 71, 393-410.